MEI

Mathematics in Education and Industry

MEI STRUCTURED MATHEMATICS

METHODS FOR ADVANCED MATHEMATICS, C3

Practice Paper C3-D

Additional materials: Answer booklet/paper
Graph paper
List of formulae (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS

- Write your Name on each sheet of paper used or the front of the booklet used.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION

- The number of marks is given in brackets [] at the end of each question or part-question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72.**
- **You are reminded of the need for clear presentation in your answers.**

Section A (36 marks)

1 You are given that $y^2 = 4x + 7$.

(i) Use implicit differentiation to find $\frac{dy}{dx}$ in terms of y. [2]

(ii) Make x the subject of the equation.

Find $\frac{dx}{dy}$ and hence show that in this case $\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$. [3]

2 **(i)** Expand $\left(e^x + e^{-x}\right)^2$. [1]

(ii) Hence find $\int \left(e^x + e^{-x}\right)^2 dx$. [3]

3 **(i)** Sketch the graph of $y = |3x - 6|$. [2]

(ii) Solve the equation $|3x - 6| = x + 4$ and illustrate your answer on your graph. [4]

4 Find $\int x \sin 3x \, dx$. [4]

5 Make x the subject of $t = \ln \sqrt{\frac{5}{(x-3)}}$. [4]

6 The function f(x) is defined as $f(x) = \frac{\ln x}{x}$. The graph of the function is shown in Fig. 6.

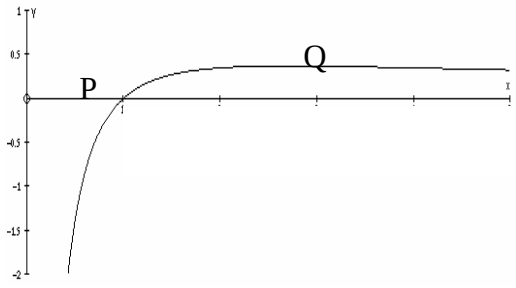

Fig. 6

(i) Give the coordinates of the point, P, where the curve crosses the x-axis. [1]

(ii) Use calculus to find the coordinates of the stationary point, Q, and show that it is a maximum. [6]

7 An oil slick is circular with radius r km and area A km^2. The radius increases with time at a rate given by $\frac{dr}{dt} = 0.5$, in kilometres per hour.

(i) Show that $\frac{dA}{dt} = \pi r$. [4]

(ii) Find the rate of increase of the area of the slick at a time when the radius is 6 km. [2]

Section B (36 marks)

8 Fig. 8 shows the graph of $y = x\sqrt{1+x}$. The point P on the curve is on the x-axis.

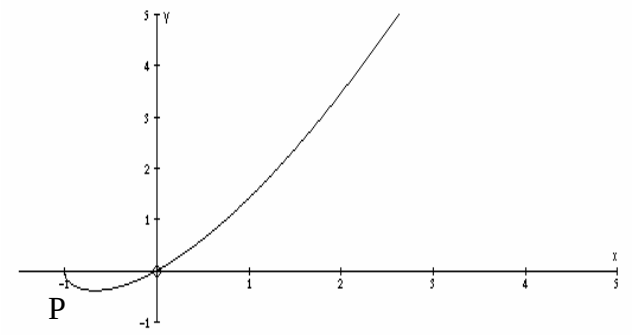

Fig. 8

(i) Write down the coordinates of P. [1]

(ii) Show that $\frac{dy}{dx} = \frac{3x+2}{2\sqrt{1+x}}$. [4]

(iii) Hence find the coordinates of the turning point on the curve.
What can you say about the gradient of the curve at P? [4]

(iv) By using a suitable substitution, show that $\int_{-1}^{0} x\sqrt{1+x}\,dx = \int_{0}^{1}\left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right)du$.

Evaluate this integral, giving your answer in an exact form.

What does this value represent? [7]

(v) Use your answer to part (ii) to differentiate $y = x\sqrt{1+x}\sin 2x$ with respect to x.
(You need not simplify your result.) [2]

9 The functions f(x) and g(x) are defined by

$$f(x) = x^2, \qquad g(x) = 2x - 1,$$

for all real values of x.

(i) State the ranges of f(x) and g(x).
Explain why f(x) has no inverse. [3]

(ii) Find an expression for the inverse function $g^{-1}(x)$ in terms of x.
Sketch the graphs of $y = g(x)$ and $y = g^{-1}(x)$ on the same axes. [4]

(iii) Find expressions for gf(x) and fg(x). [2]

(iv) Solve the equation gf(x) = fg(x).

Sketch the graphs of y = gf(x) and y = fg(x) on the same axes to illustrate your answer. [4]

(v) Show that the equation f($x + a$) = $g^2(x)$ has no solution if $a > \frac{1}{4}$. [5]